

Glide Reflections & Compositions

I can identify glide reflections in a plane.

I can represent transformations as compositions of simpler transformations.

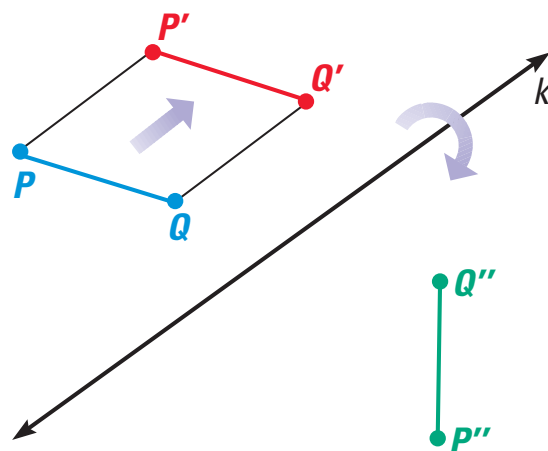
I can perform composite transformations.

THEOREM**THEOREM 7.6** *Composition Theorem*

The composition of two (or more) isometries is an isometry.

A translation, or glide, and a reflection can be performed one after the other to produce a transformation known as a *glide reflection*. A **glide reflection** is a transformation in which every point P is mapped onto a point P'' by the following steps:

1. A translation maps P onto P' .
2. A reflection in a line k parallel to the direction of the translation maps P' onto P'' .



As long as the line of reflection is parallel to the direction of the translation, it does not matter whether you glide first and then reflect, or reflect first and then glide.

When two or more transformations are combined to produce a single transformation, the result is called a **composition** of the transformations.



EXAMPLE 1 *Finding the Image of a Glide Reflection*

Use the information below to sketch the image of $\triangle ABC$ after a glide reflection.

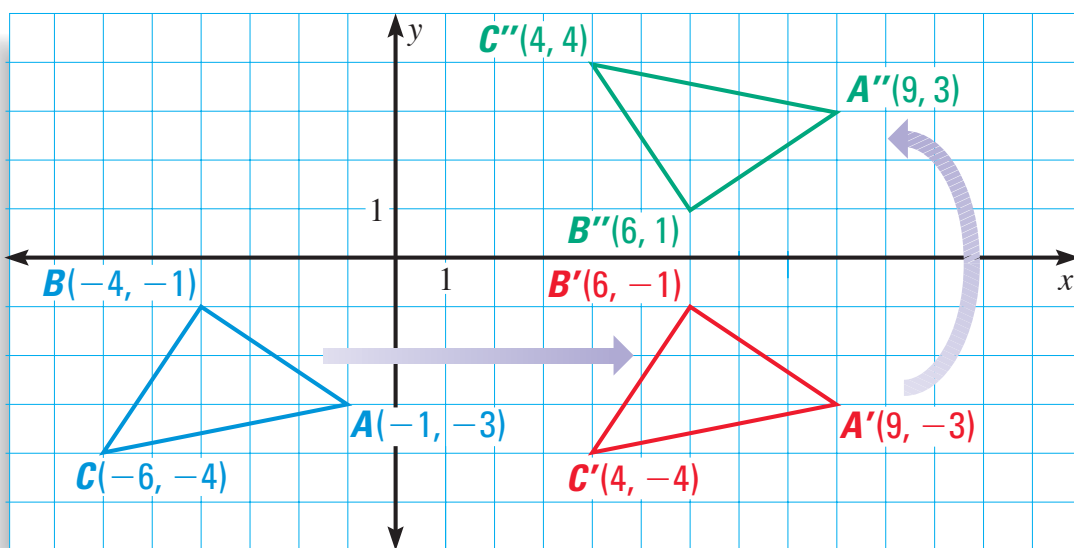
$$A(-1, -3), B(-4, -1), C(-6, -4)$$

Translation: $(x, y) \rightarrow (x + 10, y)$

Reflection: in the x -axis

SOLUTION

Begin by graphing $\triangle ABC$. Then, shift the triangle 10 units to the right to produce $\triangle A'B'C'$. Finally, reflect the triangle in the x -axis to produce $\triangle A''B''C''$.

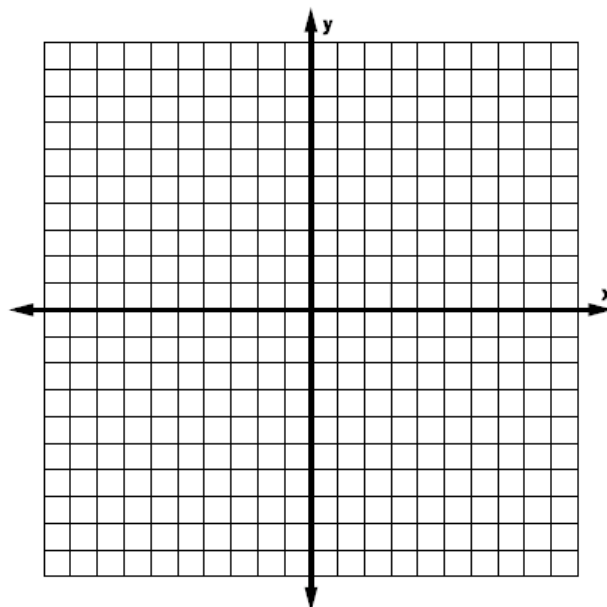


1. Use the information to sketch the image of $\triangle QRS$ after a glide reflection. $Q(2, -3)$,

$R(4, -4)$ & $S(5, -1)$.

Translation: $(x, y) \rightarrow (x + 6, y + 4)$

Reflection: y -axis.



EXAMPLE 2 Finding the Image of a Composition

Sketch the image of \overline{PQ} after a composition of the given rotation and reflection.

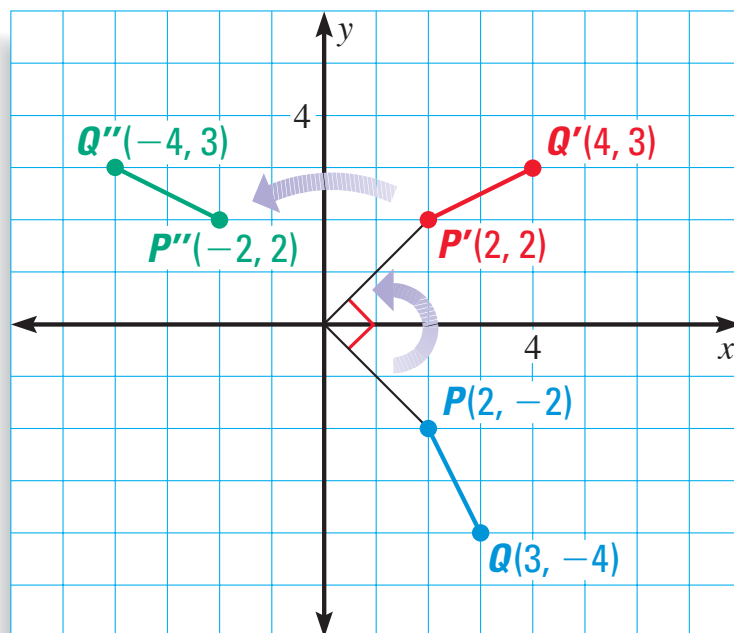
$$P(2, -2), Q(3, -4)$$

Rotation: 90° counterclockwise
about the origin

Reflection: in the y-axis

SOLUTION

Begin by graphing \overline{PQ} . Then rotate the segment 90° counterclockwise about the origin to produce $\overline{P'Q'}$. Finally, reflect the segment in the y-axis to produce $\overline{P''Q''}$.



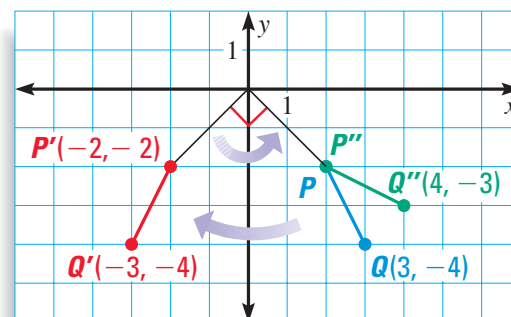
EXAMPLE 3 Comparing Orders of Compositions

Repeat Example 2, but switch the order of the composition by performing the reflection first and the rotation second. What do you notice?

SOLUTION

Graph \overline{PQ} . Then reflect the segment in the y-axis to obtain $\overline{P'Q'}$. Rotate $\overline{P'Q'}$ 90° counterclockwise about the origin to obtain $\overline{P''Q''}$. Instead of being in Quadrant II, as in Example 2, the image is in Quadrant IV.

► The order which the transformations are performed affects the final image.



STUDENT HELP

Study Tip

Unlike the addition or multiplication of real numbers, the composition of transformations is not generally commutative.

Pre-AP Geometry 7.5 Notes
Glide Reflections & Compositions

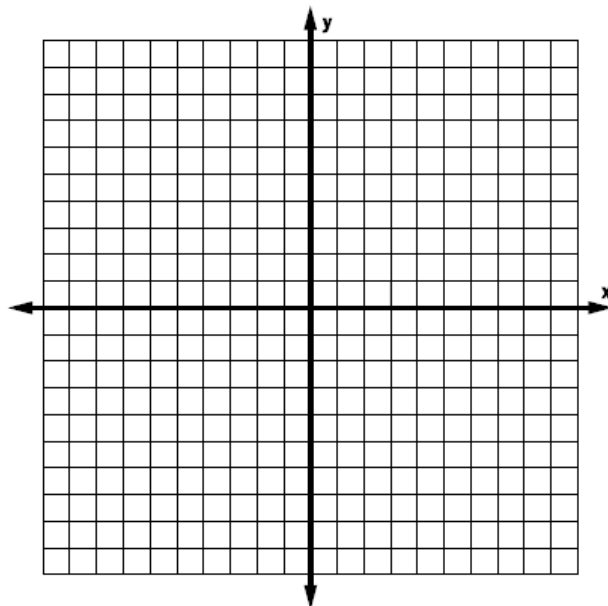
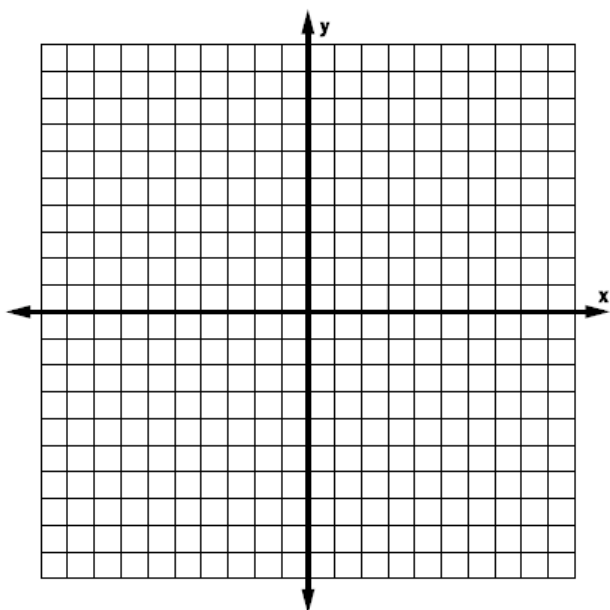
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2. Sketch the image of \overline{CD} after a composition of the given rotation and reflection. C(2, 0), D(3, 3).

Reflection: x-axis.

Rotation: -90°

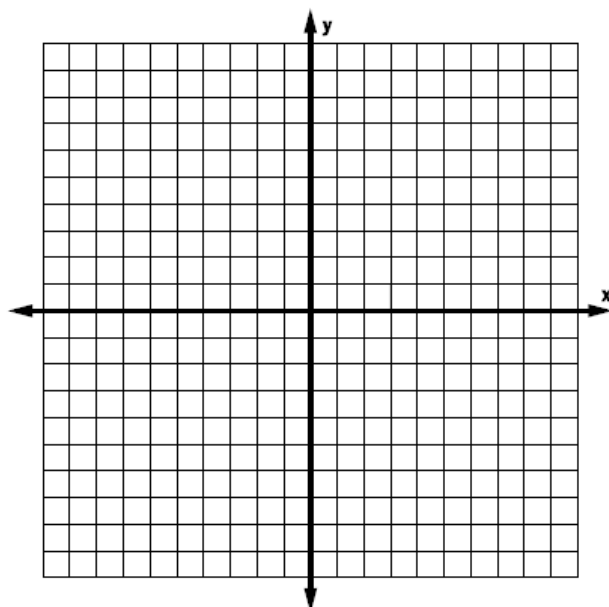
3. Repeat problem #2, but the transformations in reverse order. Are the transformations commutative?



4. State the coordinates of the image of \overline{EF} after a composition of the given rotation and translation. E(-3, 5) & F(2, 7)

rotation: 90°

translation: $(x, y) \rightarrow (x-3, y+1)$

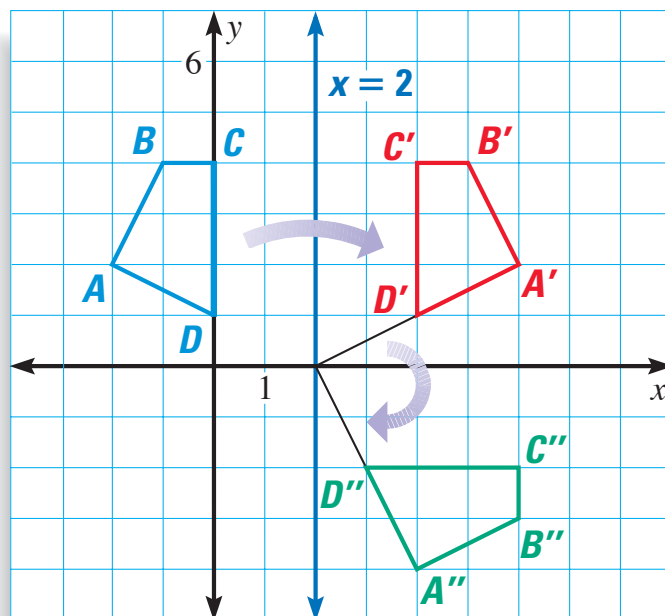


EXAMPLE 4 Describing a Composition

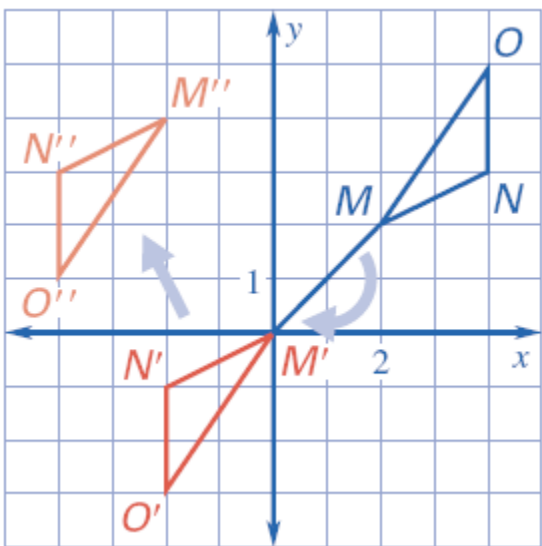
Describe the composition of transformations in the diagram.

SOLUTION

Two transformations are shown. First, figure $ABCD$ is reflected in the line $x = 2$ to produce figure $A'B'C'D'$. Then, figure $A'B'C'D'$ is rotated 90° clockwise about the point $(2, 0)$ to produce figure $A''B''C''D''$.



5. Describe the composition of transformations in the diagram.



EXAMPLE 5 Describing a Composition



PUZZLES The mathematical game pentominoes is a tiling game that uses twelve different types of tiles, each composed of five squares. The tiles are referred to by the letters they resemble. The object of the game is to pick up and arrange the tiles to create a given shape. Use compositions of transformations to describe how the tiles below will complete the 6×5 rectangle.



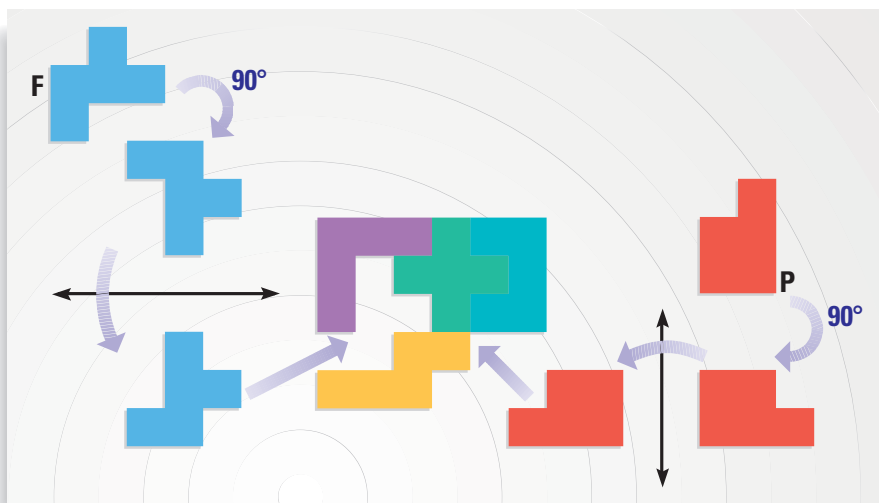
STUDENT HELP

Study Tip
You can make your own pentomino tiles by cutting the shapes out of graph paper.

SOLUTION

To complete part of the rectangle, rotate the F tile 90° clockwise, reflect the tile over a horizontal line, and translate it into place.

To complete the rest of the rectangle, rotate the P tile 90° clockwise, reflect the tile over a vertical line, and translate it into place.



CANDORVILLE

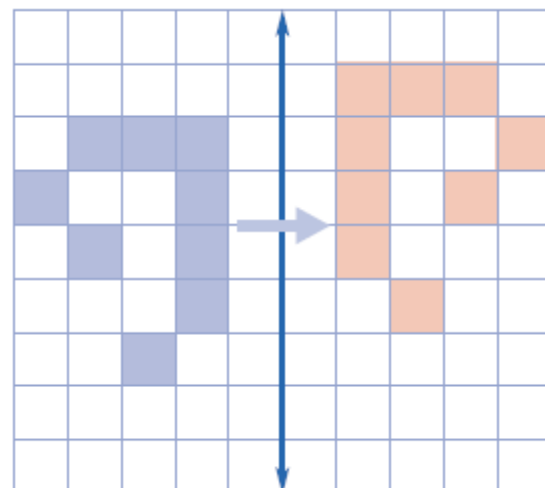
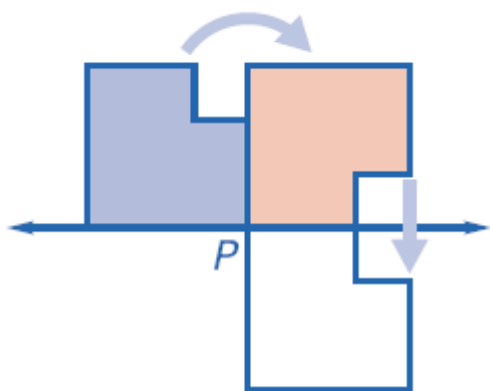
BY DARRIN BELL



Glide Reflections & Compositions

6. Describe the composition of transformations in the diagram.

7. Which series of transformations including a rotation and a reflection produced the unshaded figure from the shaded figure?



8. _____ Using the composition, what are the coordinates of the endpoints of $\overline{A''B''}$?

Given: $A(-2, 3)$, $B(3, 1)$

Reflection: $y = x$.

Rotation: -90° about $(1, 2)$

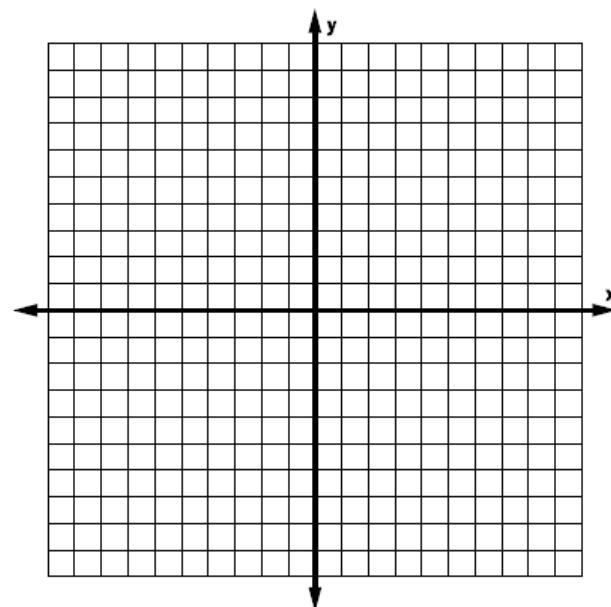
A) $A''(3, -2)$, $B''(1, 3)$

B) $A''(-3, 0)$, $B''(2, 2)$

C) $A''(0, -1)$, $B''(0, 2)$

D) $A''(5, 2)$, $B''(-2, 6)$

E) $A''(3, 0)$, $B''(2, -6)$



9. Since reflections, rotations and translations are all isometries, what can you say about a glide reflection?

10. In a glide reflection, the direction of the _____ must be parallel to the line of _____.

Glide Reflections & Compositions

Complete the statement with *always*, *sometimes*, or *never*.

11. The order in which two transformations are performed ____? ____ affects the resulting image.

12. In a glide reflection, the order in which the two transformations are performed ____? ____ matters.

13. A composition of isometries is ____? ____ an isometry.

